Between Philosophy and Mathematics: Examples of Interactions in Classical Islam

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In this paper the author raises the question of the existence and the extension of philosophy of mathematics in classical Islam. He considers three types of interactions between mathematics and theoretical philosophy. The first is that used by al-Kindī who uses the means and methods of mathematics to reconstruct his philosophical system. The second type emerges when the mathematician al-Ṭūsī tries to solve a philosophical question: the emanation of the multiplicity. The third type is comes from the mathematician al-Sijzi, who philosophically solves a mathematical problem but does not yet have the means to do so mathematically.

Keywords: Philosophy and mathematics in Islamic tradition; links between philosophy and science; history of Islamic science; al-Kindī; al-Ṭūsī; al-Sijzi; Islamic scientific tradition; science, religion and philosophy.

Historians of Islamic philosophy have a keen interest in what is often readily called *falsafa*. According to their understanding, it deals with the doctrines of Being and the Soul, as developed by the authors of the Islamic culture, without any consideration for other fields of knowledge and independent of all determination, if it were not for the link these doctrines may have with religion. Thus, Islamic philosophers are perceived to be evolving within the Aristotelian tradition of neo-Platonism, and to be no more than heirs of late Antiquity, albeit with an Islamic 'touch'. This kind of historical understanding appears to assure us

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that from the ninth century onwards, the passage of Aristotle, Plotinus, and Proclus, among others, to the Islamic philosophers had taken place safely, without the slightest trouble. But this historical account is advanced not without some serious consequences: not only does it paint for us a very pale and an impoverished picture of philosophical activity, it also transforms the historian into an archaeologist whose skills he happens to lack. Indeed, it is not at all rare that historians set themselves the task of rummaging through the terrain of Islamic philosophy in search of vestiges of Greek works whose originals are lost but are thought to be preserved in the Arabic translations, or that, for lack of better resources, they are at least satisfied with the traces of writings of the philosophers of Antiquity, often studied with the competence and talent characteristic of the historians of Greek philosophy. It is in this manner that the history of Islamic philosophy is transformed into archaeology, so to speak.

It is true that aside from the Greek heritage some historians have recently turned their attention to doctrines developed in other disciplines, such as philosophy of law, masterly developed by the jurists; philosophy of *kalām,* which reached important depths and stages of refinement; the Sufism of the great masters like al-Ḥallāj and Ibn ʿArabī, and so on. There is no doubt that studies of this sort go a long way towards enriching and rectifying our vision and better reflecting the philosophical activity of the time.

Science and mathematics, however, are far from having received the same favourable attention given to law, *kalām,* linguistics, or Sufism. Furthermore, the examining of the relationships, which we regard as essential, between the sciences and philosophy, and notably between mathematics and philosophy, is completely disregarded. This serious lacuna is not the sole responsibility of the historians of philosophy; it is that of the historians of science as well.

It is true that the examination of these relationships requires various competencies and an in-depth knowledge of these areas: in addition to a linguistic knowledge which goes beyond that called for in geometry (there one might do with elementary syntax and a poor lexicon), one is also asked to have a grasp of the history of philosophy itself. If we realize these requirements are not often met, and on top of that the conception of the relationships between science and philosophy is inherited from an oft-ambient positivism, we may come to better appreciate the profound indifference exhibited by the historians of science vis-à-vis the kind of examination we are calling for. This is despite the well-known fact that the links which science and philosophy enjoy are but an integral part of the history of sciences.

As a matter of fact, the situation is quite paradoxical: while scientific and mathematical research of the most advanced standards had been developed and worked out in Arabic in the urban centres of the *Islamicate* for a period of seven centuries, is it at all conceivable that philosophers, who were often themselves mathematicians, doctors, and so forth, would have remained recluse in their philosophical activity, totally oblivious to the mutations taking place under their eyes, and completely blind to the successive scientific results that were then being achieved? Moreover, when faced with such an outburst of new disciplines and also success (an astronomy critical of Ptolemaic models, optics reformed and renewed, an algebra created, an algebraic geometry invented, a Diophantine analysis transformed, a theory of the parallels discussed, projective methods developed, and so on) can it be imagined that philosophers remained unperturbed by these developments, as to deduce that they were strictly confined to the relatively narrow frame of the Aristotelian tradition of neo-Platonism?

The seemingly impoverished philosophy in the classical period of Islam, which emerges as a result of such accounts, is due to historians rather than history.

Actually, to examine the relationships between philosophy and science, or between philosophy and mathematics—which is our only concern here— as they appear in the works of the 'pure' philosophers, amounts to only a third of the quest desired here. There is a further need to address in this instance, and that is to bring in the views of both the mathematician-philosophers and the 'pure' mathematicians. But why have we from the beginning taken the view that we should limit our consideration in this instance to mathematics? This idea deserves an explanation, particularly as this approach is by no means exclusive to Islamic philosophy.

No other scientific disciplines have contributed to the genesis of theoretical philosophy as mathematics have, and none other than mathematics have established links with philosophy which are not only numerous but also ancient. Indeed, since antiquity, mathematics had never ceased to present to the philosophers' reflection central themes of vital importance: it is owing to mathematics that philosophers acquired their methods of exposition, their argumentation procedure, and were provided even with instruments which they appropriated for their analysis. Mathematics are themselves presented to the philosopher as an object of study, when the latter devotes himself to the clarification of mathematical knowledge, studying its object, methods, whilst inquiring on the characters of its apodictic. Throughout the history of philosophy, questions about this mathematical knowledge (its genesis, its power of extension, the nature of certitude it attains, its place in the classification of knowledge) have been asked relentlessly. In that regard, philosophers of Islam during the classic period are no exception to the rule: al-Kindī, al-Fārābī, Avicenna, Ibn Bājja, Maimonides—to name only a few—are no exception to the rule.

Links other than these have also been established between mathematics and theoretical philosophy, albeit in a more subtle and discrete fashion. Whether the aim is forging a method, even logic (as in the meeting of Aristotle and Euclid in relation to the axiomatic method), or the appeal of al-Ṭūsī's combinatory analysis in order to solve the philosophical problem of the emanation from the one, their collaboration is again not rare. There is, however, in all those possible forms of relationship, one that is of a particular significance, and one which is instigated by the mathematician and not the philosopher: those doctrines mathematicians worked out in order to justify their own practice. The most opportune conditions for such theoretical constructions are gathered at the time when the mathematician spearheading the research of his period is hindered by an unsurmountable difficulty: namely, when the available mathematical techniques are deemed inadequate in the face of the emerging and spreading new objects. One could think of the diverse variants from the theory of parallels, particularly from the time of Thābit ibn Qurra, to the sort of *analysis situs* conceived by Ibn al-Haytham, for the doctrines of the invisibles in the seventeenth century.

The relationships between theoretical philosophy and mathematics are essentially found in three types of intellectual oeuvre: those of the philosophers; those of the philosophers-mathematicians like al-Kindī, Naṣīr al-Dīn al-Ṭūsī, etc.; and those of the mathematicians of the avant-garde, such as Thābit ibn Qurra, his grandson Ibrahīm ibn Sinān, al-Qūhī, Ibn al-Haytham, and so on. If we restrict ourselves to this group or the other when examining the relationships philosophy and mathematics enjoy with one another, we are bound to lose an essential dimension in this domain.

We have endeavoured on many occasions to exhibit some of the themes of this philosophy of mathematics simply by digging here and there, our aim being to unearth samples demonstrating the richness of this domain rather than its systematic examination, which is not our purpose here. We do recognize that such a project deserves quite a large book. Yet, what seems most suitable to us is to move away from the pure relation of the views, which philosophers may have expressed on mathematics and their importance; our way is mostly interested in the themes that were debated, in the intimate relationships that unite mathematics to philosophy, and their role in the propping up or scaffolding of doctrines or systems. We are, in other words, seeking the organising role of mathematics. We will first highlight how the philosopher-mathematicians proceed in their search for mathematical solutions to philosophical problems, a fruitful approach which generates new doctrines and even new disciplines. As we look at the mathematicians we will bring into relief their attempts at solving philosophically mathematical problems. It will become clear that such an intellectual enterprise is indeed necessary and of far-reaching consequences.

To clarify the aforementioned typology, I will now mention some examples in brief.

I. Al-Kindī as Philosopher-Mathematician

An appreciation of the relationships between philosophy and mathematics is essential for the reconstitution of al-Kindī's system. Is it not this very dependence which drives the philosopher to write a book entitled *Philosophy Can Only be Acquired Through the Mastery of Mathematics* and an epistle by the title *On the Quantity of Aristotle's Works,* in which he presents mathematics as a foundational course prior to the teaching of philosophy? In that epistle he even goes so far as to call the student of philosophy, warning him that he is in fact before the following alternative: either to start with the study of mathematics prior to embarking on the works of Aristotle, as grouped and ordered by al-Kindī, and then the student could hope to become a real philosopher—or else he could skip the study of mathematics altogether, and be only an imitator, so long as his memory does not fail him in this task. It is clear that for al-Kindī mathematics form the very base of the philosophical course. If we were to delve into their role in the philosophy of al-Kindī—which is not what we are doing here—we could then grasp more rigorously the specificity of his oeuvre. Historians tend to look at al-Kindī's philosophical edifice under two clearly distinct lights. According to the first interpretation, al-Kindī comes across as a Muslim who represents the Aristotelian tradition of neo-Platonism—that is, a philosopher of a doubly-late antiquity. The second sees in him a continuator of speculative theology (*kalām*)—that is, a theologian who had to switch languages in order to speak of Greek philosophy. But if we were to grant mathematics the true role they play in the elaboration of philosophy, then al-Kindī's fundamental options would be brought to relief: one of them stems from his Islamic convictions, as they are explicated and formulated in the tradition of the speculative theology, particularly that of *al-Ṭawḥīd*, which holds that revelation brings us the truth to which reason can attain; the other falls back and refers to Euclid's *Elements* as a model and as method: the rational can also be attained by way of the truths inherent to reason, which need to comply with the criteria of the geometric proof, and are therefore independent of revelation. These truths of reason, which here serve as primitive notions and postulates, are, during the period of al-Kindī, brought about by the Aristotelian Tradition of neo-Platonism. It is these truths that are then chosen to replace the truths offered by revelation to speculative theology, so long as they can satisfy the exigencies of geometric thinking and allow for an exposition that is axiomatic in all appearances. It is in this way that 'the mathematical examination' (*al-faḥṣ al-riyāḍī*) has become the instrument of metaphysics.

Epistles in theoretical philosophy such as *First Philosophy* as well as *On the Finitude of the Universe* are in fact a case in point. In relation to the latter, for instance, al-Kindī proceeds in an orderly manner in order to demonstrate the inconsistency of the concept of the infinite body. There he begins by defining the primitive terms: magnitude and homogenous magnitudes. He then introduces what he calls an "absolute/assertoric proposition" (*qaḍiyya ḥaqq*), or as he explains elsewhere, "first premises, true and intelligible through immediate inference" (*al-muqaddimāt al-uwal al-ḥaqqiyya al-maʿqūla bi-lā tawassuṭ*), or, again, "first premises, evident, true and immediately intelligible" (*al-muqaddimāt al-ūla al-wāḍiḥa al-ḥaqqiyya al-maʿqūla bi-lā tawassuṭ*); that is to say tautological propositions. These are formulated in terms of primitive notions, relations of order thereof, operations of reunions and separation thereof, and predications: finite and infinite. It is to do with propositions such as of homogenous magnitudes, the magnitudes of which are not greater than the others are equal; or, if to one of the equal homogenous magnitudes is added a magnitude homogenous to it, the magnitudes would be unequal. Finally, al-Kindī proceeds by demonstration, through *reductio ad absurdum*, using a hypothesis: the part of an infinite magnitude is necessarily finite.

II. Nāṣir al-Dīn al-Ṭūsī as Mathematician-Philosopher (or A Mathematical Resolution of a Philosophical Question)

In this example, we are called to reflect on other relationships between mathematics and philosophy in classical Islam: the ties that are established between the two in the instance when the philosopher borrows an instrument from mathematics with the aim of solving a logico-metaphysical question. Nevertheless, the situation which is of particular interest to us here has a specific trait: this borrowing returns dividends to the mathematical domain which provided the instrument, enhancing its progress and advancement. The exchange between combinatorics and metaphysics is an excellent illustration of this double movement: did not Ibn Sina give, on the basis of his ontological and cosmogonical conceptions, a formulation of the doctrine of the emanation from the One? And did not al-Ṭūsī, when endeavouring to derive multiplicity from the One, manage to see that the doctrine as developed by Ibn Sīnā could still be dotted with a combinatorial armature, which was borrowed then from the algebraists? For al-Ṭūsī's move to work, however, the rules of combination of the algebraists had to be interpreted in a combinatorial manner. And it is this very combinatorial interpretation that finally marked the birth of this discipline called combinative analysis, much exploited after al-Ṭūsī by mathematicians like al-Fārisī and Ibn al-Bannāʾ, among others. On the basis of this contribution, al-Ḥalabī, a philosopher of a later period, will attempt to organise the elements of the new discipline, assigning to it a name as a way of demarcating it and underscoring its autonomy.

It is imperative for us to, nonetheless, distinguish this movement from the kind of progression like that of Raymond Lulle. Lulle has combined notions following mechanical rules, the results of which appeared later to be arrangements and combinations. But Lulle has borrowed nothing from mathematics and his approach does not have the slightest consideration for mathematics. This is unlike al-Ṭūsī whose approach comes very close to that of Leibniz, despite the differences that may exist between the two projects: the first, as we have already stated, intends to solve mathematically the issue of the emanation of the multiplicity from the One, which in the end brought him to galvanise the Avicenian doctrine of creation with a combinatorial armature; the second wanted in fact to build an *ars inveniendi* on the combinatorial.

The emanation of the intelligences, of the celestial bodies as well as the other worlds—that of nature and that of corporeal things—from the One, is one of the central doctrines of Avicenna's metaphysics. This doctrine raises a question which is ontological and noetic at the same time: how is it that from one being, unique and simple, there could emanate a multiplicity, which is also a complexity that comprises in the end both the matter of things and the form of the bodies and the human souls? This ontological and noetic duality raises an obstacle, as both a logical and metaphysical difficulty that must be disentangled. From this we understand, at least in part, why Ibn Sīnā time and again had returned to this doctrine, and implicitly to this question.

A study of the historical evolution of Ibn Sīnā's thought on this issue, in light of his different writings, would reveal to us how he managed to amend an initial formulation with such a difficulty in mind. Let us confine our field to his *al-Shifa*ʾ and *al-Ishārāt wa al-Tanbīhāt*. There Ibn Sīnā exposes the principles of this doctrine as well as the rules of the emanation of multiples from a simple unity. His explication seems articulate and orderly, but is short of constituting a rigorous proof: in fact, he does not provide there the syntactical rules apt to espouse the semantic of the emanation, while it is precisely in this that the difficulty of the question related to the derivation of the multiplicity from the One resides. Indeed, this issue of derivation had been perceived as a problem for a long time. The mathematician, philosopher, and commentator of Ibn Sīnā, Naṣīr al-Dīn al-Ṭūsī (1201-1273), not only grasped the difficulty, but he also wanted to come up with the syntactical rules which were then lacking.

In his commentary on *al-Ishārāt wa al-Tanbīhāt*, al-Ṭūsī introduces the language and the procedures of combination to follow the emanation until the third order of beings. He then stops the application of these procedures to conclude: "if we were to exceed these three orders [the first three], there may exist an uncountable multiplicity (*lā yuḥṣā ʿadaduhā*) in one single order, and to the infinite". Al-Ṭūsī's intention is thus clear, just as the procedure he applied for the first three orders leaves us with no doubt: the proof and the means that Ibn Sīnā lacked must be put forward. At this stage, however, al-Ṭūsī is still far from the aim. It is one thing to proceed by combinations for a number of objects, and it is another thing altogether to construct a language with its syntax. Here, this language would be that of combinations. And it is precisely for the introduction of this language that al-Ṭūsī strives in a separate essay, whose title leaves no ambiguity in the air: *On the Demonstration Concerning the Mode of the Emanation of Things in Infinite [Numbers] from the Beginning of the First and Unique Principle*. In this instance, al-Ṭūsī proceeds generally through the combinatorial analysis. Al-Ṭūsī's text and the results it contains will not disappear with their author; they are found again in a later treatise which was completely devoted to the combinative analysis. Thus, not only does al-Ṭūsī's solution mark out a style of research in philosophy, it also represents an interesting contribution to the very history of mathematics.

III. Al-Sijzī as Mathematician (or A Philosophical Solution to a Mathematical Problem)

In proposition fourteen of his book *On the Conics,* Apollonius proposes to demonstrate that the asymptotes and the hyperbole come closer to one another indefinitely without actually ever meeting. This proposition obviously calls for that formidable notion of the infinite. Firstly, the infinite is presented as an object of knowledge, since what is at issue are mathematical beings whose existence entail infinite processes. This is but a trait peculiar to any asymptomatic behaviour. The idea of the infinite, however, also comes to the fore as a means for knowledge called for by the infinite mathematical construction, such as the infinite construction of the rest of the distances between the curve and its asymptote: one needs to ascertain that it is always possible to reiterate the same construction.

Now, it is not hard to understand that this notion of the infinite, as professed by Apollonius, must have bothered both mathematicians and philosophers. For if the former could not have remained indifferent to an obvious difficulty in the demonstration, mainly due to the use of a notion which was never clearly drawn, the latter must have been attuned to a new problem which happens to be emerging at that very conjuncture, and whose traces had continued to persist even during the eighteenth century viz., the gap between our ability to conceive of a property and our capacity to actually rigorously establish it. Can we establish a mathematical property which we can not distinctly conceive? We may need to take a few steps back in order to localise the commencement of this philosophical interrogation.

In the eyes of al-Sijzī, this proposition covers up a problematic which he tries to elicit in the rather inflected language of Islamic Aristotelian philosophy. In line with the Aristotelian Islamic philosophers, al-Sijzī seems in fact to admit that mathematical knowledge, like any other knowledge, may be characterised by the twosome "conception/judgment" (*taṣawwur/taṣdīq*); whereas in mathematics this twosome is confined to that of "conception/demonstration", in that here judgment is considered but a demonstrative syllogism. Again in line with the Aristotelians, al-

Sijzī only recognizes that conception which is 'essential' and revealed by way of a rational intuition or expressed in a definition. In his case, as in others indeed, we may paraphrase that famous text of the *Second Analytics*: conception "shows what a thing is, whereas demonstration shows whether a thing is or is not attributed to a given other". Following this terminology, Apollonius' proposition raises the problem of those affirmations which are demonstrable while they remain inconceivable or hardly conceivable at the very least.

Having said that, we know that to establish the proposition of Apollonius rigorously one needed to make use of concepts and techniques which al-Sijzī as a mathematician had not yet possessed: these are the concepts and the means for the analysis. But, in this case, it is worth noting that philosophical elucidation allows the mathematician to actually elbow in and make a dent till the plotting of future mathematical pathways is laid out. And if the mathematical difficulty calls for a philosophical thematic, philosophical explication is in turn presented as a means for the reflection of the mathematician. It is these two tasks together which completely characterise al-Sijzī's approach.

In the first instance, he is driven to make a comparison between conception and demonstration with the aim of establishing a typology of mathematical propositions, which will then permit him to close in on the exact type of Apollonius proposition. Following the Aristotelian philosophers, he begins by recognising two extreme types, the confrontation of which shows clearly that there can not be a conception of all that lends itself to demonstration: this is precisely the case with the proposition advanced by Apollonius. We could, on the other hand, seize the essence of the object of a proposition, and conceive of it without having recourse to a demonstration. Between these two extreme types are the others, the intermediary ones; and it is then that al-Sijzī brings to relief the classification of the mathematical propositions.

1. Propositions which are directly conceivable on the basis of philosophical principles.
2. Propositions which are conceivable prior to attempting their demonstration.
3. Propositions which are conceivable only once an idea of their demonstration is formed.
4. Propositions which are conceivable only once they are demonstrated.

5. Propositions which are hardly conceivable, even after they had been demonstrated.

Thus, al-Sijzī has provided us with one of the earliest classifications of mathematical propositions from the twosome “conception/ demonstration”.

IV. The Philosophical Problem of the Unity of Mathematics: the Mathematicians’ Examination

Let us recall, although it goes without saying, that the heirs of Hellenic mathematics had been accumulating results and methods through active research for more than two centuries, and were thus led to conceive of disciplines unknown to the Greeks: algebra, the entire Diophantine analysis, and the algebraic theory of the cubic equations, to name only a few. Again, with their access to works in astronomy, optics, and statistics, mathematicians were also led to ‘renew’ Hellenic geometry, introducing new chapters to it. Among the renewed disciplines, one could list infinitesimal geometry, spheric geometry, and so on. As for the new chapters, they deal with geometry of position and form, particularly on the study of geometrical transformations.

The language of the quadrivium had proven inept to contain such diversity, and things had already got a bit jammed with that of the theory of proportions. Consequently, a search began for another mode of demonstration that could be algebraic and undertake projections both through proportions and folding-backs. It still remained that the global landscape—the development of an increasing diversity on the one hand and the persistence of a wooden language on the other—was begging, as it were, for both a logical examination and a philosophical elucidation. Philosophers of the calibre of al-Fārābī appear to have anticipated some of the difficulties engendered by this situation. The latter had for instance conceived a new ontology of the mathematical object and an architecture other than that of the quadrivium, for the purpose of the composition of an encyclopaedia of mathematics and of other forms of knowledge in general. However, for reasons that were at once theoretical as well as practical, it behoved the mathematicians alone to face to these difficulties, and indeed, soon thereafter they came up against them, especially in their compositions on the analysis and the synthesis. This encyclopaedic aspect of the analysis and the synthesis has since recalled a vibrant problem, but one obscured in this context: to render an account of the new disciplines, and restore the unity of mathematics.

By the end of the ninth and beginning of the tenth century, the term 'mathematic' and the term 'geometric' pertained to a host of dispersed disciplines, which from then onwards could be contained by the increasingly narrow frame of the quadrivium. As a matter of fact, it is no longer possible to gather all of these disciplines under one denomination, like that of the 'theory of the magnitudes', for instance. How is the unity of mathematics thought out in these conditions? This is a question at once necessary and difficult: there had been no means at the time, and for a long time still, to attain this unity. Algebra was still far from being the discipline of the algebraic structures, and was not formalised at all. It could only do some partial unifications, such as the geometry of conics and the theory of equations. As the algebra of structures was yet to be created, mathematicians had no other choice but to find another way: the idea was to find a discipline that was logically prior to all of the other mathematical disciplines, but which did not predate them historically, and was necessarily posterior to all of them, so that it was able to actually provide them with the unifying principles. In the meantime, no determination of the nature of this discipline or its methods and objects was *a priori* called for. The analysis and the synthesis had manifestly played the role of this unifying discipline. Ibn Sinān (909-946) does not preoccupy himself with the entire field of mathematics, but only with geometry, although the unification he did is brought to bear on the gamut of the analysis and synthesis procedures, and the reasonings deployed, independently of the realm of geometry in which they apply. The discipline which justifies the method, viz. the analysis and synthesis as a discipline, is a sort of programmatic logic, in that it allows one to associate an *ars inveniendi* to an *ars demonstrandi*.

Ibn Sinān's contribution is of a particular interest: it is the first substantial essay, to our knowledge, which deals with this kind of philosophical logic of mathematics. The author has thus brought the fundamental problem of the unity of geometry to this logico-philosophical discipline of the analysis and the synthesis, inaugurating in this way an entire tradition that can be traced throughout the tenth century all the way to the algebraist al-Samawʾal in the twelfth century. Indeed, it is also following Ibn Sinān and against him that Ibn al-Haytham would develop his project.

With Ibn Sinān we cannot say that we have reached the middle of the great century when the mathematical activity is at its zenith. The differentiation between the disciplines was, nonetheless, following its course; geometry of projections had received a strong jolt at the hands of mathematicians like al-Qūhī and Ibn Sahl; geometrical transformations had become an object of reflection and application for the mathematicians; a chapter on geometrical constructions with the help of the conics had taken shape and developed. For geometrical demonstrations we now increasingly have recourse to a folding-back, to punctual transformations, and to the asymptotic properties of conical curves to demonstrate their points of intersection. Simply put, two types of exigencies had then emerged: demonstrative structures have to be conceived for the new objects as well as providing for their plan of existence. It was recognized the accomplishment of these two intimately linked approaches also requires that the methods employed would have to be founded on the basis of a discipline. This would have to also be general enough as to be able to offer the means of existence to the new geometrical objects, without being reduced to a pure logic; but additionally, it would have to precede logically all the other mathematical disciplines in order to provide foundations to the diverse demonstrative structures. It is this monumental task that Ibn al-Haytham has tackled, no doubt by choice, but by necessity as well. We owe to him the undertaking of innovative research of the highest degree not only in all the branches of geometry, but also in arithmetic and the Euclidean theory of numbers, and these are precisely the domains that would occupy him the most.

We have indicated above four situations in which philosopher-mathematicians, mathematician-philosophers, and 'pure' mathematicians delved into the topic of the philosophy of mathematics, and brought forth as much evidence as is possible for the blossoming of this field from the ninth century onwards. To forget these contributions not only impoverishes the history of philosophy, but it goes as far as to truncate the history of mathematics as well.

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